PART IB EXPERIMENTAL ENGINEERING

SUBJECT: INFORMATION ENGINEERING EXPERIMENT I3 (SHORT)

LOCATION: EIETL, INGLIS BUILDING

POSITION CONTROL

Objectives

1. To give experience of interpreting block diagrams.
2. To relate control system behavior to *natural frequency* and *damping* of 2nd-order differential equations.
3. To introduce cascade control.
4. To give experience of making frequency response measurements (including phase shifts) at very low frequencies (if time allows).
5. To give experience of designing a control system to meet specifications.

Introduction

In this experiment you will control the position of a steel ball on a beam by means of negative feedback. The acceleration of the ball will be determined by the angle of the beam from the horizontal. The experiment consists of three parts: (1) Design of a feedback system to control the beam angle; (2) Design of a feedback system to control the ball position, making use of the control system designed in (1)(so-called *cascade control*); (3) Measurement of the frequency response (harmonic response) of the ball position control system.

The behaviours you will observe, and the calculations you will perform, are typical of those encountered in the control of many electromechanical systems such as active vehicle suspensions, compact disc players, robots, aircraft, and many others.

Near the back of the lab sheet you will find the figures, and a page on which you should record the results you obtain.

Equipment

1 unit comprising:

Ball and beam with motor and ball position sensor.
Beam position and velocity sensors mounted on motor shaft.
'Patch panel' with 4 variable-gain amplifiers ($P1, P2, Q1, Q2$), motor drive amplifier, signal limiter(not used), 'demand signal' generator, position and velocity signal output sockets($BP, -BV, PP, -PV$).
Note: Output sockets are red, input sockets are green.
Low frequency signal generator.
Voltmeter(± 10 volts).
Data Logger Interface connected to a PC for two-channel data logging; this allows both 'Y versus time' and 'Y versus X' plots to be made, as well as having a 'DVM' (Digital Volt Meter) facility.
Flying leads(for patching the 'patch panel' etc).

Note that the position and velocity signals for the beam are prefixed with 'P'(= 'Plank'), and those for the ball are prefixed with 'B'. Whenever we refer to these signals we shall assume that they have already been transduced into electrical signals, measured in volts. Some of these signals have minus signs attached to them, to indicate that they have been inverted by the electronics before being brought to the patch panel.

Also note that socket labeled '$-BV$' provides a 'Ball Velocity' signal, even though there is no direct measurement of this. This signal is derived from the Ball Position signal; for the purposes of this experiment treat it as if it were a direct measurement.

Design of Beam Control System

Remove the ball from the beam

The angle of the beam will be controlled by a feedback system of the form shown in fig.1 PD is the *Plank Position Demand* (electrical) signal, which represents the angle we would like the beam to have. PP is fed back compared with PD, any error is amplified by the gain of K_2, and fed into the input of the drive amplifier(labeled $DRIVE$). The sign of the feedback is chosen such that the motor is driven to reduce the error. There is little inherent damping in the motor/beam system, so feedback of PP alone would lead to a very underdamped control system. We therefore also feed back PV(which has been amplified internally by the gain R before reaching the patch panel), amplify it by the gain K_1, and add it to the position error signal before applying it to $DRIVE$. (This is known as 'velocity feedback' for obvious reasons).

Using the lower two potentiometers on the 'patch panel', implement the beam angle control system shown in fig.1. Use potentiometer $Q1$ to implement K_1 and $Q2$ to implement K_2. (Don't worry about the potentiometer settings yet.)

The motion of the beam is described approximately by

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = a \times DRIVE \tag{1}$$

where θ is the beam angle(in radians), J is the beam's moment of inertia, B is a viscous damping term, and a is the effective torque constant of the motor and drive amplifier. Since PP is measured with sensitivity of 0.67 volts/deg, we can rewrite (1) as

$$\frac{d^2(PP)}{dt^2} + \beta\frac{d(PP)}{dt} = \alpha \times DRIVE \tag{2}$$

where

$$\alpha = 0.67 \times \frac{180}{\pi} \times \frac{a}{J} \quad \text{and} \quad \beta = \frac{B}{J} \tag{3}$$

When the feedback loops shown in fig.1 have been closed, it is straightforward to show that

$$\frac{d^2(PP)}{dt^2} + (\beta + \alpha R K_1)\frac{d(PP)}{dt} + \alpha K_2(PP) = \alpha K_2(PD) \tag{4}$$

The auxiliary equation (or characteristic equation) of (4) is

$$s^2 + (\beta + \alpha R K_1)s + \alpha K_2 = 0 \tag{5}$$

(in Control engineering we usually use s rather than λ, etc, because it is also used as the complex variable appearing in the Laplace Transform).

If K_1 and K_2 are chosen so that 5 has two coincident real roots located at σ, ie

$$s^2 + (\beta + \alpha R K_1)s + \alpha K_2 \equiv (s - \sigma)^2 \tag{6}$$

then the response of PP to a unit step on PD will be critically damped, with time constant $1/\sigma$:

$$PP(t) = 1 - (1 - \sigma t)e^{\sigma t} \tag{7}$$

(Note that σ must be negative if the system is to be stable.)

Assuming $\alpha = 30$, $\beta = 0$, and $R = 0.15$, what values of K_1 and K_2 are required to obtain critical damping with $\sigma = -20$?

Set up the potentiometers $Q1$ and $Q2$ to give these values of K_1 and K_2. (Remember to take account of the $\times 20$ amplifier!)

Note that if the potentiometer 'window' is showing '2', say, then its gain is between 0.2 and 0.3, depending on the dial setting. That is, the potentiometers have a gain of 1.00 when fully open.

Record the response of the beam angle control system to step changes of PD of about 1 volt. If necessary, adjust $Q1$ to obtain approximately critical damping.

Note: Instructions for using the *Data Logger Interface* are attached at the back of this lab sheet.

If necessary, adjust $Q2$ and $Q1$ until the beam step response is substantially finished in about 0.5 second or less, with damping approximately critical.

Ball Position Control

The Ball Position will be controlled by a feedback system as shown in fig.2. BD is an electrical signal which represents that *Ball Position Demand.* The ball position measurement BP is fed back and compared with BD, any error is amplified by the gain K_4, and used to change the beam angle (PP) to correct the ball position. Of course, in order to change PP the error signal must be used as the input to the beam control system which you have already built. Thus PD is no longer obtained from a signal generator, but from the Ball Position signal. As before, velocity feedback is used to control the damping of the ball control system: BV is fed back, amplified by the gain K_3, and combined with the ball position error signal, as shown in fig.2.

Implement the feedback system shown in fig.2, using the potentiometers $P1$ and $P2$ to implement the gains $K3$ and $K4$, respectively. (Leave your beam control system as it is, since you will need it as part of the ball control system.)

Set $K_3 = 0.1$ and $K_4 = 0.8$. (You are advised to set the the appropriate amplifier gain to '$\times 2$'. Remember to take this amplifier into account.)

The ball control system should now be stable, but very underdamped. You are going to design it so that it can meet a fairly tough specification. In order to do this, we have to look at a little theory.

If x is the position of the ball on the beam then, for small beam angles,

$$\ddot{x} = \frac{g}{1.8}\theta \tag{8}$$

(where g is the acceleration due to gravity, and the derivation of the term $g/1.8$ is quite complicated). Since the ball position is measured with a sensitivity of 20 volts/metre, we have

$$\frac{d^2BP}{dt^2} = \gamma \times PP \tag{9}$$

where $\gamma = (g/1.8) \times (20/0.67) \times (\pi/180) = 2.84$.

Now if you have implemented the beam angle control system correctly, then the response of PP to changes in PD is going to be much faster than the response of the ball to changes in BD. We shall therefore make the simplifying assumption that

$$PP = PD \tag{10}$$

namely that the beam angle responds instantly to demands, (whereas the real relationship between PD and PP is given by equation (4)). With this assumption, it is easy to show that, when the feedback loops shown in fig.2 have been closed,

$$\frac{d^2(BP)}{dt^2} + \gamma K_3 \frac{d(BP)}{dt} + \gamma K_4(BP) = \gamma K_4(BD) \tag{11}$$

From the left hand-side of this, we get the auxiliary equation

$$s^2 + \gamma K_3 s + \gamma K_4 = 0 \tag{12}$$

Comparing 12 with the standard form for a second-order system:

$$s^2 + 2 * \zeta\omega_n s + \omega_n^2 = 0 \tag{13}$$

where ω_n is the *natural frequency* and ζ is the *damping factor*, we see that

$$K_3 = \frac{2\zeta\omega_n}{\gamma} \quad \text{and} \quad K_4 = \frac{\omega_n^2}{\gamma} \tag{14}$$

Fig.3 shows how the step response of a second-order system varies with the damping factor ζ. Note that the horizontal axis is 'normalized time' $\omega_n t$, so that changing the undamped natural frequency ω_n changes the speed with which the response occurs, but leaves its shape unchanged.

Keeping K_4 unchanged, record the step response of the ball with two different values of K_3, and verify that the damping is increased as K_3 is increased.

Now increase K_4 by a factor of 2, and K_3 by a $\sqrt{2}$. According to 14 this should leave the damping factor unchanged, but increase the undamped natural frequency by a factor of $\sqrt{2}$. Record a step response and verify this.

Now design the ball control system to meet the following specification: The ball is required to follow a square-wave demand signal, of frequency 0.1Hz, and the error between BP and BD should be less than 20% of the demand amplitude for at least 70% of the cycle. It is suggested that you perform the design as follows:

Choose a damping factor ζ between 0.5 and 1.0 (choose one of these shown on fig.3). For this ζ, determine from fig.3 the value of ω_n required to meet the specification, assuming that the response during each half-cycle is the same as it would be for an isolated step (which is not quite true). Determine the values of K_3 and K_4 required to obtain these values of ζ and ω_n.

Implement these values of K_3 and K_4.

Using the signal generator, apply a square wave ball demand signal of amplitude 4 volts and frequency 0.1 Hz, and record the response of the ball

Check whether your design meets the specification

Frequency response measurements (if you have time)

Equation (11) can be re-written as a *transfer function* between BD to BP:

$$\frac{BP(s)}{BD(s)} = \frac{\gamma K_4}{s^2 + \gamma K_3 s + \gamma K_4} \tag{15}$$

and from this we can obtain the *frequency response* at frequency ω by replacing s by $j\omega$. This means that if $BD(t) = X(\omega)\sin(\omega t)$ then $BP(t) \to Y(\omega)\sin(\omega t + \phi(\omega))$ as $t \to \infty$, where the *gain* or *amplification* is

$$\frac{Y(\omega)}{X(\omega)} = \left| \frac{\gamma K_4}{(j\omega)^2 + \gamma K_3 j\omega + \gamma K_4)} \right| \tag{16}$$

and the *phase shift* is

$$\phi(\omega) = \arg\left(\frac{\gamma K_4}{(j\omega)^2 + \gamma K_3 j\omega + \gamma K_4)}\right) \tag{17}$$

In particular if we take $\omega = \omega_n = \sqrt{(\gamma K_4)}$ then

$$\frac{Y(\omega_n)}{X(\omega_n)} = \frac{K_4}{K_3\omega_n} = \frac{1}{2\zeta} \quad \text{and} \quad \phi(\omega_n) = -\frac{\pi}{2}(\text{rad}) \tag{18}$$

Note that for $\omega \ll \omega_n$ we have $Y(\omega)/X(\omega) \approx 1$ and $\phi(\omega) \approx 0$, whereas for $\omega \gg \omega_n$ we have $Y(\omega)/X(\omega) \approx \gamma K_4/\omega^2$ and $\phi(\omega) \approx -\pi$. In the latter case we therefore have

$$\log\frac{Y(\omega)}{X(\omega)} \approx -2\log\omega + \log(\gamma K_4) \tag{19}$$

so that the graph of $\log[Y(\omega)/X(\omega)]$ against $\log\omega$ (ie a Bode plot) is almost a straight line at high frequencies.

A convenient way of measuring the frequency response is to apply BD to channel A of the *Data Logger Interface*, and apply BP to channel B. Once initial transients have died away, and elliptical display should be obtained. From this the gain(Y/X) and phase (ϕ) characteristics can be obtained, as shown in fig.4

Apply a sinusoidal ball position demand (BD) signal of 2 volts (peak to peak), at the predicted undamped natural frequency ω_n of your ball control system (remember to convert rad/sec to Hz). Check whether the phase shift at the frequency is $-\pi/2$ rad.

Adjust the frequency of BD until the phase shift is $-\pi/2$, and hence determine the actual resonant frequency of your system. By also measuring the gain at this frequency, estimate the actual damping ζ (using equation (18)).

If time permits, measure the gain and phase shift at frequencies $\omega_n/5$, $\omega_n/2$, $2\omega_n$, and $5\omega_n$ (using the experimentally determined value of ω_n), and plot results on the graph paper provided.

Michaelmas 2015/Lent 2016

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Answer Sheet

Design of Beam Control System

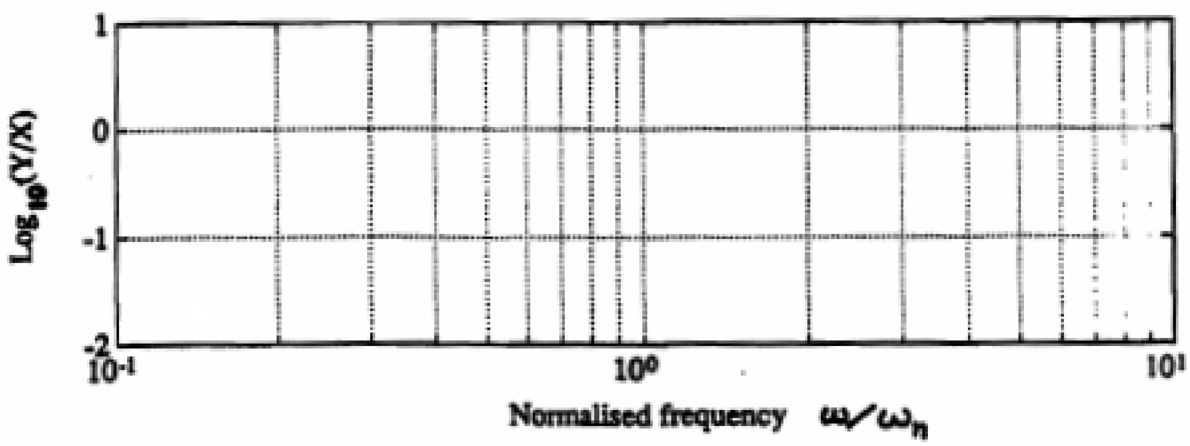
1.	Values of K_1 and K_2 for $\sigma = -20$	$K_1 =$	$K_2 =$
	Potentiometer settings:	$Q_1 =$	$Q_2 =$
2.	Potentiometer setting needed for critical damping:	$Q_1 =$	
3.	Final potentiometer settings for beam control system:	$Q_1 =$	$Q_2 =$

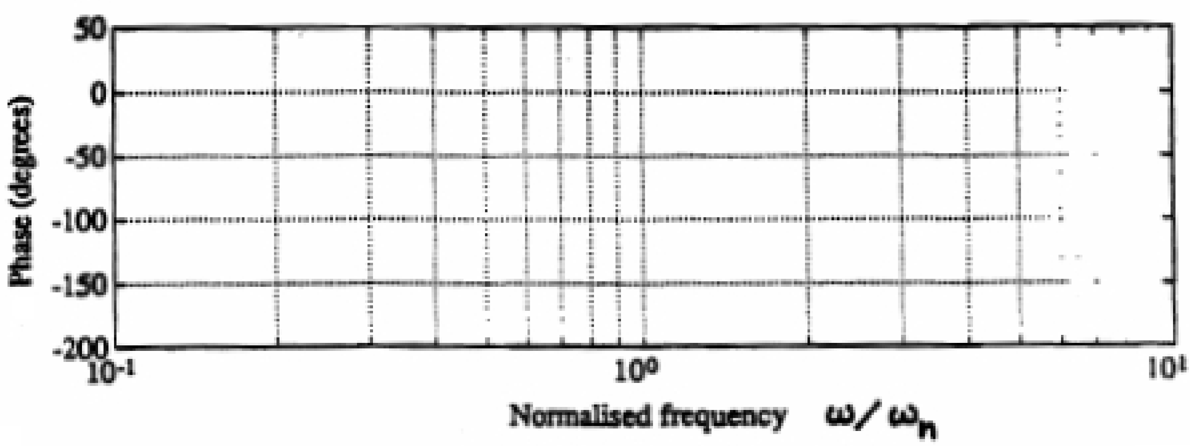
Ball Position Control

4.	Damping factor and natural frequency selected:	$\zeta =$	$\omega_n =$
5.	Corresponding values of K_3 and K_4:	$K_3 =$	$K_4 =$
6.	Proportional of cycle for which error $< 20\%$:		
7.	Final potentiometer settings:	$P1$	$P2$

Frequency response measurements

8.	Phase shift at designed ω_n :	ϕ
9.	Frequency at which $\phi = -\pi/2rad$:	ω_n
10.	Actual damping at this frequency:	ζ





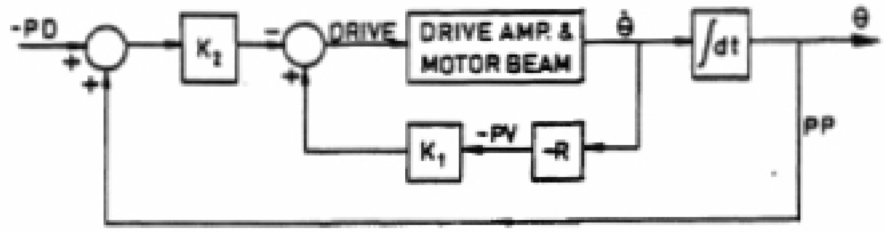


Fig. 1

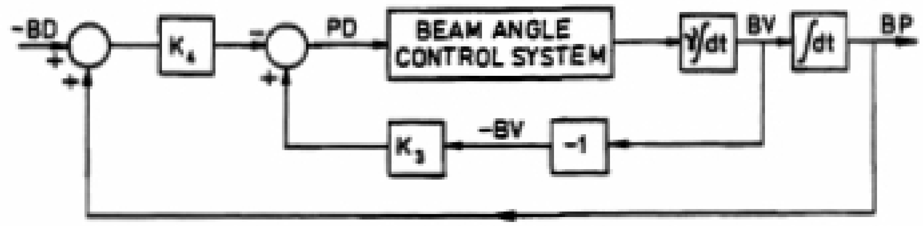


Fig. 2

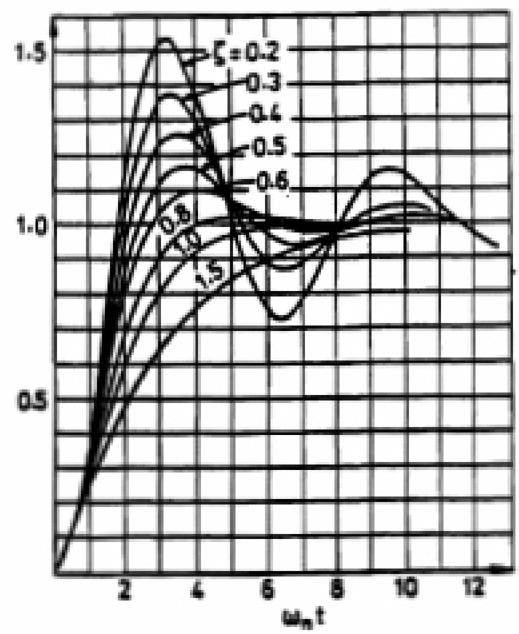


Fig. 3 Step Response of 2nd Order System

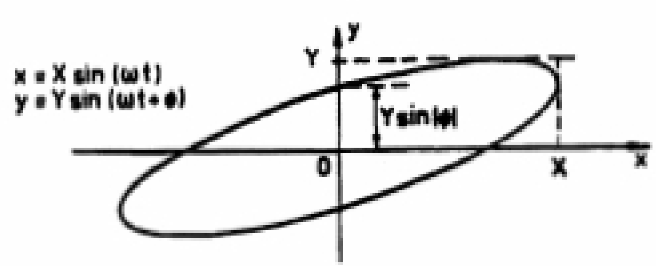


Fig. 4 Assumes that the sensitivities of both channels are equal.

The Data Logger Interface