

# PART IB EXPERIMENTAL ENGINEERING

SUBJECT: INFORMATION ENGINEERING  
EXPERIMENT: I1 (long)  
LOCATION: EIETL, INGLIS BUILDING

## SPECTRUM ANALYSIS AND AMPLITUDE MODULATION <sup>1</sup>

### 1 Objectives

1. To introduce the concept of spectral analysis.
2. To become acquainted with the use of a simple computer-based spectrum analyser.
3. To measure the spectra of various simple waveforms.
4. To study the spectra of amplitude modulated signals and characteristics of a simple demodulator.

### 2 Introduction

The characteristics of electrical (and other) signals can be explored in two ways. The first is to examine their waveforms in the time domain, which can be done using an oscilloscope. The second is to examine their spectra in the frequency domain, which can be done using a *spectrum analyser*. Both descriptions are useful, although frequency domain analysis is the most natural tool for many tasks in electrical circuit design, speech and music analysis, and for video signals. In fact, frequency domain analysis is of fundamental importance in all branches of engineering, from the design of bridges and roads to wireless communications.

This laboratory introduces the concepts of spectral analysis and analyses the frequency spectra of some simple waveforms using a computer-based digital spectrum analyser. Spectrum analysers are often not computer-based. However, the use of a computer allows for easy long-term storage and display of results as well as greater flexibility and off-line data manipulation capability. The spectrum analyser used here can also be used as an oscilloscope.

### 3 Spectrum Analysis

Spectrum analysis attempts to identify the relative quantities of different frequencies which are present in a given signal. Take, for example, a simple signal  $x(t)$  made up of two pure sine-waves with frequencies 1 rad/sec and 2 rad/sec, respectively:

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<sup>1</sup>Handout updated on 06/9/2015 by Ramji Venkataramanan.

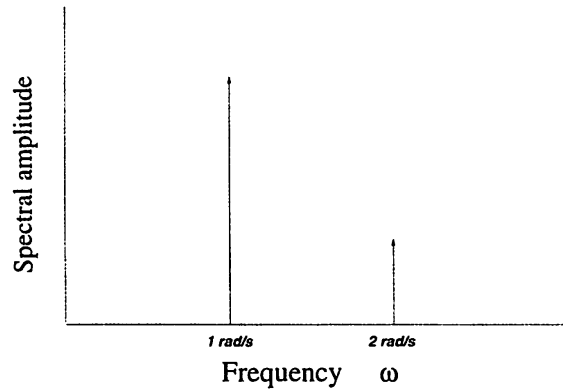


Figure 1: Amplitude spectrum of  $3 \sin(t) + \sin(2t)$

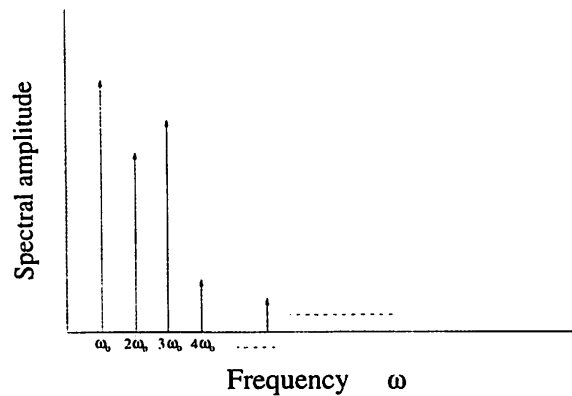


Figure 2: Amplitude spectrum of signal with period  $T$

$$x(t) = 3 \sin(t) + \sin(2t).$$

There are exactly two ‘frequency’ components present in this signal: 1 rad/sec and 2 rad/sec. The ‘amplitude’ of the first frequency component is three times that of the second. We can represent this information in a plot of amplitude against frequency—a frequency amplitude spectrum. In this case, the spectrum will be zero at all frequencies except 1 rad/sec and 2 rad/sec. In order to represent the pure sine wave components we use impulse or ‘ $\delta$ -functions’<sup>2</sup> centred on 1 rad/sec and 2 rad/sec, weighted by the amplitudes 3 and 1 respectively, as in Figure 1.

We have already seen a closely related analysis in the Fourier series representation of a periodic signal with period  $T$ :

$$x(t) = a_0 + \sum_{n=1} \{a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\},$$

where  $\omega_0 = \frac{2\pi}{T}$ .

Since the mean-squared value of each term  $a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$  is proportional to  $\sqrt{a_n^2 + b_n^2}$ , we can represent the spectrum of  $x(t)$  as a series of  $\delta$ -functions, centred on frequencies  $n\omega_0$  and weighted by the corresponding terms  $\sqrt{a_n^2 + b_n^2}$ , as in Figure 2.

Both of these are examples of *line spectra*, because the spectra of pure sines and cosines (‘sinu-

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<sup>2</sup>Recall that  $\delta(x - x_0)$  takes the value zero at all points except  $x = x_0$ , and that its total area is equal to unity.

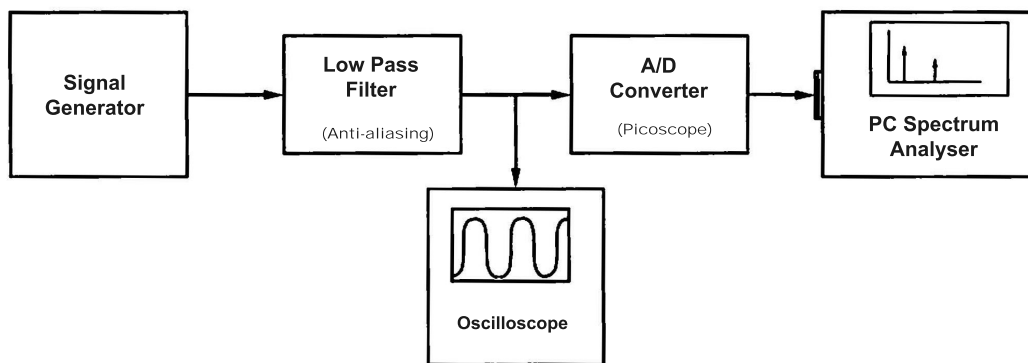


Figure 3: Block Diagram of Apparatus

soids’) are a-functions, or spectral ‘lines’. The theory of frequency spectra is more general than this, allowing for signals which are not made up of pure sinusoids. In this practical, however, we will be examining only signals which can be represented as sums of sinusoids and hence lead to line spectra. In the Lent Term, lectures cover the complete theory, known as the *Fourier Transform*.

A spectrum analyser is a piece of equipment which takes as its input the signal  $x(t)$  and automatically calculates the frequency spectrum. This is then usually displayed on a screen (in our case the screen of a computer), as described in the next section.

## 4 The Computer-Based Spectrum Analyser

The spectrum analyser comprises a low-pass (‘anti-aliasing’) filter and an analogue to digital converter (*Picoscope*) which interfaces directly to the parallel port on a PC (the software in this laboratory is set up to run under Windows 7). The anti-aliasing filter limits the range of frequencies which are input to the computer. The spectral analysis is all performed on the PC by the *Picoscope* software. Details of the software operation can be found in the appendix at the end of this lab sheet, and there is also a manual available in the laboratory. A block diagram of the apparatus is given in Figure 3.

The full theory of both analogue and digital spectral analysis will be studied in the Signal and Data Analysis course (Lent Term) under the topics of the Fourier Transform (FT) and Discrete Fourier Transform (DFT). We do not require the detailed theory for this practical. However, a useful way to think of a spectrum analyser (and indeed a possible way of constructing an analogue spectrum analyser) is as a large bank of frequency selective filters. Each filter is centered upon a particular value of frequency  $\omega$  and allows a very narrow range of frequencies around  $\omega$  to be passed through to its output. The amplitude of the output is then approximately proportional to the frequency spectrum at  $\omega$ . A more practical alternative is to use a single narrow band filter whose centre frequency is rapidly swept over the required range of frequencies to determine the spectrum.

## 5 Spectra of Simple Periodic Signals

1. You are provided with a simple signal source which can generate a number of different waveforms. Connect the output of this to the anti-aliasing filter input, switch the source to position 4 (sine wave) and set the frequency to 2kHz. With the oscilloscope and the spectrum analyser connected as shown in Figure 3 and the *Picoscope* spectrum analyser running, you should be able to measure the frequency of your sine wave both on the oscilloscope and on the PC. For

ease of viewing, set the *Picoscope* program to display the frequency spectrum over a range around 0 – 4 kHz or 0 – 8 kHz (you can set this using the drop-down menu). You should get a display corresponding to one single frequency component at 2kHz. Record the spectrum you observe. There is also an oscilloscope function within the spectrum analyser (click on the ‘scope mode’ button); it is useful to display both this and the frequency plot.

2. Experiment with displaying different frequency ranges on the *Picoscope* and varying the amplitude/frequency of the input signal. Check that the frequency spectra agree with the frequencies measured on an oscilloscope. You should be familiar with the basic software functionality by this stage. You should also be convinced that it is easier to measure frequencies via the spectrum analyser software than via the oscilloscope and that the software oscilloscope is easier to use.
3. The Fourier series expansion of a perfect square wave with peak-to-peak amplitude 2 and period  $T$  is given by:

$$z(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

where  $\omega_0 = \frac{2\pi}{T}$  and

$$b_n = \begin{cases} \frac{4}{\pi} \frac{1}{n}, & n = 1, 3, 5, \dots \\ 0, & \text{otherwise} \end{cases}$$

Plot the relative Fourier coefficient  $b_n/b_1$  against  $n$  for  $1 \leq n \leq 10$ .

4. Set the signal generator to output a square wave (position 5). Observe and sketch the waveform and adjust the signal amplitude to 2.0V peak-to-peak, and the period to 1ms (i.e., 1kHz frequency). Measure the spectrum. Measure the spectral amplitude of the first 10 harmonics and plot the relative amplitude on the same graph as the theoretical values. Do your measurements agree with the theory? If you repeat this exercise without the low-pass/anti-aliasing filter in the circuit, do you see any differences in the spectrum?
5. Repeat tasks 3 and 4 for a triangular wave with the same frequency and amplitude. Plot all your results. The theoretical coefficient values in this case are:

$$b_n = \begin{cases} \frac{8}{\pi^2} \frac{(-1)^{(n-1)/2}}{n^2}, & n = 1, 3, 5, \dots \\ 0, & \text{otherwise} \end{cases}$$

Since a single triangular pulse can be formed by the convolution of two single rectangular pulses, we can form a triangular wave by convolving a square pulse with a square wave. Using this fact, but without detailed calculations, verify that we should expect the Fourier coefficients for the triangular wave to go like  $1/n^2$ .

## 6 Amplitude Modulation and Demodulation

In communications channels such as wireless radio it is necessary to multiplex many sets of information (e.g. different radio stations) into the same channel. A common means of achieving this is by amplitude modulation (AM).

Consider an information signal which is a pure cosine wave:  $x(t) = E \cos(\omega t)$ . Now generate a ‘carrier’ signal  $x_c(t) = E_c \cos(\omega_c t)$ . The modulated signal is generated by modulating the amplitude

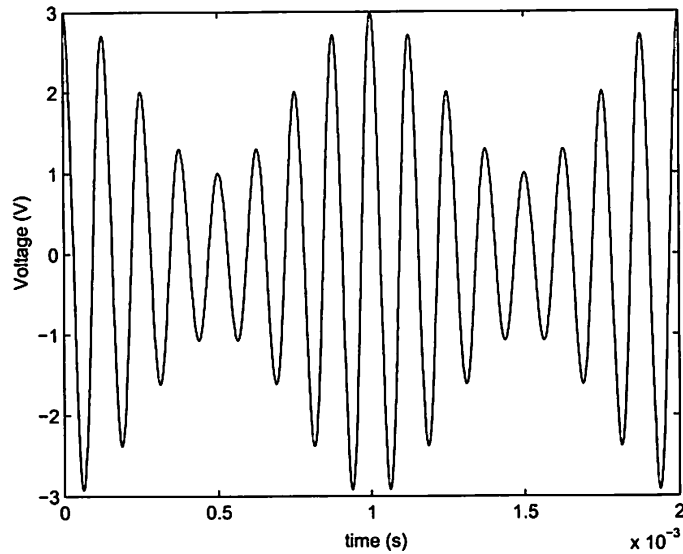


Figure 4: Modulated Waveform

of the carrier signal by the information signal as follows:

$$x_m(t) = E_c(1 + m \cos(\omega t)) \cos(\omega_c t), \quad (1)$$

where  $m = E/E_c$  is known as the *modulation index*. Expanding (1) gives,

$$x_m(t) = E_c \cos(\omega_c t) + \frac{mE_c}{2} \cos((\omega_c + \omega)t) + \frac{mE_c}{2} \cos((\omega_c - \omega)t). \quad (2)$$

This expression shows clearly that the modulated signal has three frequency components: one at the carrier frequency and two sidebands at  $\omega_c + \omega$  and  $\omega_c - \omega$ .

1. How is this useful in transmitting many band-limited signals over a common medium?
2. Now we set up the circuit to generate a carrier signal (which is preset at about 8kHz — check the exact frequency of the carrier via the spectrum analyser) which can be modulated via other signals from the generator. Set the signal source to position 1 which generates a sine wave locked to 1/8 of the carrier frequency (i.e. 1kHz). Look at the modulated output on the software oscilloscope and adjust the carrier amplitude and modulation index until the display appears similar to Figure 4. Record both the signal and the spectrum corresponding to the signal.
3. What is the modulation index in this case? (Note that the ratio of the maximum amplitude of the envelope to the minimum value of the envelope is  $\frac{1+m}{1-m}$ )
4. What level of  $E_{min} = \min\{E_c(1 + m \cos(\omega t))\}$  corresponds to  $m = 1$ ?
5. Adjust the carrier level and modulation depth so that  $m = 1$  and record the waveform and its spectrum.
6. In all the above cases there was a component of the spectrum at the carrier frequency. However, by setting the carrier level to its minimum we can reduce this component to zero, which gives

a suppressed carrier signal. Do this, and note the waveform and the spectrum—in particular, look at how the waveform differs from that when  $m = 1$ . What is the value of  $m$  in this case?

7. Re-adjust the modulator to give a waveform similar to Figure 4. Connect the modulated output to the input of the demodulator circuit. Make sketches of the waveform and spectra of the signals available at the input and output as well as the two intermediate testing points TP1 and TP2 of the demodulator box.
8. By considering the recorded waveforms and spectra, design a simple circuit involving basic components to achieve the operation of the demodulator, indicating the input, the output, and the testing points TP1 and TP2. State suitable values for various parameters of the circuit (such as cut-off frequencies). Explain using simple diagrams the operation of this circuit in the time and frequency domains.

## 7 Report

Since this is a long experiment, you need to write a report. Refer to the handout you received at the beginning of the year for guidelines. In particular, you should include:

- Summary: A brief summary of the aims of the experiment.
- Readings and Results: Include all the measurements asked for during the experiment.
- Discussion: Discuss the results and answer the questions posed in the text.
- Conclusions: Summarise your findings and achievements.

**IMPORTANT:** Please sign up for a marking session for **I1 Spectrum Analysis** at:  
<http://to.eng.cam.ac.uk/teaching/apps/cuedle/listofbookingsheets.php>



## Appendix: Using the Software

The experiment uses a particular type of datalogger called a *Picoscope*, with software running under Windows 7. The *Picoscope* acts as a digital oscilloscope and spectrum analyser. The software is easily operated and its basic functionality will be detailed by the demonstrator at the beginning of the lab session. If the PCs are switched on and running under Linux, restart the machine and during boot-up, choose the Windows option.

Log on using your Teaching System ID and password.

**Starting up:** Connect a sine wave signal to channel 1 of the Picoscope box, as is indicated in Figure 3. Double click on the Picoscope 6 shortcut and the program will start. The Picoscope panel will open; you should also see the sine wave live as on an oscilloscope.

**Triggering:** What you will see now is probably a very unstable looking sine wave. To make the waveform stable you need to set up the triggering of the Picoscope. Triggering refers to the idea that the Picoscope should display the waveform so that it always goes through a certain voltage e.g. 0 volts at a certain point in time. Then, if the waveform is periodic it will appear stationary on the screen providing the frequency is high enough. To set up the triggering, click on the on the Trigger Mode box near the bottom left of the Picoscope screen, and set the triggering to Auto to give a stable waveform. If all is well, this should be a sine wave. Notice that a small yellow diamond has appeared on the screen. This tells the Picoscope that the waveform is to be displayed such that it always goes through the yellow diamond, thereby keeping the appearance of the waveform stable.

**Spectrum Mode and Scope Mode:** Next click on the Spectrum Mode button of the panel; a new window with the spectrum analyser will be initialised. You can set the frequency range using the drop-down menu. Set the range to around 0 – 24 kHz, and experiment with varying this range. You can go back and forth viewing the waveform in time and frequency domains by clicking on the Scope Mode and the Spectrum mode buttons, respectively. You can use the zoom button to increase the precision of your readings.

If you wish to find out the full scope of its capabilities, manuals for the Picoscope are available in the laboratory or online at <http://www.picotech.com/picoscope2000-docs.html>.