PART IB EXPERIMENTAL ENGINEERING

SUBJECT: ELECTRICAL ENGINEERING EXPERIMENT E4 (LONG)
LOCATION: ELECTRICAL & INFORMATION ENGINEERING LABORATORY

WAVE TRANSMISSION & REFLECTION

These experiments involve properties of waves common to sound waves in air, seismic waves in solids, radio and optical waves in free space or in solid refracting material, and (with some reservations) gravity waves on the surface of liquids. The experiment is performed with microwaves, meaning electro-magnetic waves with wavelengths of a few centimetres, which are convenient for measurement. Microwaves are used for line-of-sight communications including satellite communications, mobile telephones, satellite navigation, radar, industrial processing, and cooking.

Objectives

To introduce some general properties of travelling waves, and the effect of combining waves travelling in opposite directions – which produces standing waves.

To measure the wavelength and velocity of microwaves in air by observing standing waves in a coaxial line.

To introduce, and emphasise, the importance of quantitative assessment of accuracy and consistency, for *all* your results in whatever experiment. The recommended approach is described in the pamphlet: *Estimation of Experimental Errors*, attached at the end of this sheet, and you should read it carefully before performing the experiment.

To measure the velocity of microwaves in some solid material (e.g. teflon, perspex and glass) from the change in travel time when the material is substituted for air, and to characterise the material by its refractive index, n.

To build a microwave resonator, by making a section line with highly reflective ends. To plot and interpret the response versus frequency for resonators of two different lengths.

1. Introduction

At optical wavelengths it is a matter of common experience that very well-defined beams of radiation can be produced in free-space propagation. For microwaves such narrow beams are not normally possible and so, in this experiment the waves are guided by two conductors with the electromagnetic fields confined within a coaxial line, or more loosely restrained by two flat conductors: a 'stripline'. The latter configuration makes it easier for us to introduce different materials into the wave-field space to observe their effect.

Some common characteristics of all waves are briefly reviewed below.

1.1 Waves have frequency, f, wavelength, λ, and velocity, u (more strictly we should say 'phase velocity') which are related by:

$$u = f\lambda \quad (1)$$

For sound waves in air u = 330 m/s whereas in steel u = 5000 m/s. Gravity waves on the surface of water generally travel at a few m/s but are dispersive, which means the velocity varies with frequency.

For EM waves in a vacuum the velocity is a universal constant denoted by $c = 2.99 \times 10^8$ ms^{-1}. It is hardly any different in air, but slower in solids and liquids and characterised by the refractive index of the material, n, where

$$u\ (\text{material}) = \frac{c}{n} \quad (2)$$

1.2 Waves also have amplitude, A. Suppose a refers to the wave variable: for sound it might be the air pressure or the air particle displacement. For EM waves it might be the electric or the magnetic field strength. Then at a <u>fixed position</u> the time-variation can be written

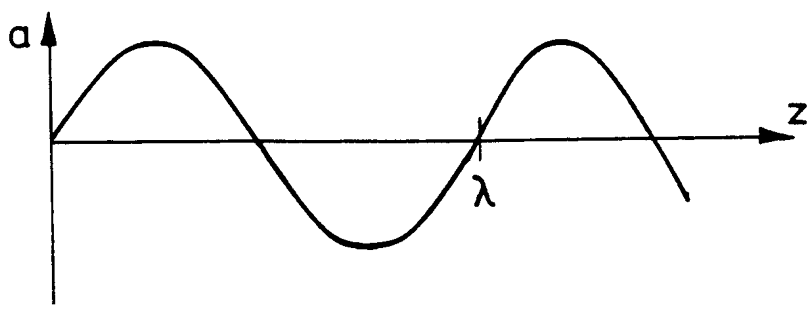
$$a = A \sin 2\pi ft \quad (3)$$

where A is the amplitude. The phase changes by 2π, so that the wave repeats itself, in a period $t = 1/f$.

Considering the variation with position z, at a <u>fixed instant</u> of time, we can write

$$a = A \sin 2\pi(z/\lambda) \quad (4)$$

so that the wave repeats itself at intervals of λ on the z-axis.



1.3 Combining (3) and (4), for a forward-travelling wave we have

$$a = A \sin \frac{2\pi}{\lambda} (ut - z) \tag{5}$$

The term 'phase velocity' can now be understood: for an observer who moved with velocity u in the positive z-direction, (i.e. $z = z_0 + ut$) then the phase term $(ut - z)$ would be unchanged.

Waves may travel in either direction so for a wave in the negative z-direction we can put

$$b = B \sin \frac{2\pi}{\lambda} (ut + z) \tag{6}$$

assuming a and b are in phase at $z = 0$.

1.4 Most changes of the material of propagation or the local geometry of the structure which supports the wave, will cause a reflection of the wave, in general a partial reflection (eg the change from tube to open air reflects the wave at the end of an organ pipe, forming a resonator). The amplitude reflection coefficient is defined by

$$\rho = \frac{B}{A} \tag{7}$$

Since the power carried by a wave is proportional to amplitude squared, the power reflection coefficient is

$$\frac{P_b}{P_a} = \frac{B^2}{A^2} = \rho^2 \tag{8}$$

This would usually be in optical systems where 'intensity' = power per unit area.

1.5 Reflections can be reduced, where desirable, by using absorbing or matching elements: 'absorbing' as in acoustic tiles, and 'matching' as in the anti-reflection coating on a lens surface. The design of these elements involves the concept of wave impedance. For light waves, and for sound waves, the impedance of solids and liquids is invariably lower than the impedance of air. In section 3.2 you will measure typical reflection coefficients.

1.6 The interference between a forward and reflected wave is due to the superposition of the a and b fields leading to 'standing waves' (fixed in space) with *maxima* where a and b add, and *minima* where they subtract because they are in anti-phase. If the amplitudes A and B are nearly equal the minima are nearly zero and are known as 'nodes'.

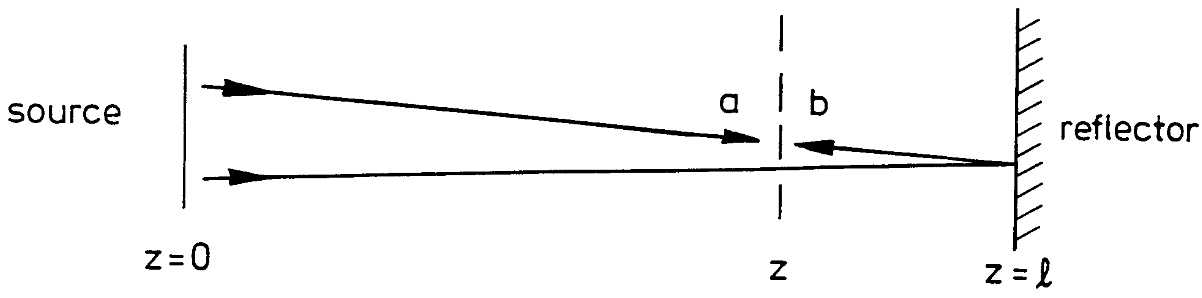


Fig. 1

Consider the superposition of the direct wave *a* and the reflected wave *b* in Fig. 1. At any z-position, *a* is given by equation (5) and looking at the distance travelled by the reflected wave in Fig. 1, we have

$$b = B \sin\frac{2\pi}{\lambda}\left[ut - (2\ell - z)\right] \qquad (9)$$

Superposing the two waves in equation (4) and (9), *a* and *b* are in phase when

$$\left[ut - (2\ell - z)\right] = (ut - z) - m\lambda, \qquad m = \pm \text{ integer}$$

i.e.

$$z_{max} = \ell - m(\lambda/2) \qquad (10)$$

For each value of m we have a maximum standing wave amplitude = ($A + B$).

For minima, *a* and *b* are in anti-phase, where the amplitude = ($A - B$), and clearly the spacing of successive maxima or minima is $\lambda/2$. In this experiment we shall make measurements on the position of minima since they can be observed using an oscilloscope or datalogger using its most sensitive scale, and consequently their position can be more sharply defined than the maxima.

Standing waves as treated in this experiment are an example of a one-dimensional interference pattern when two or more coherent waves are superposed. What other examples of interference patterns have you encountered?

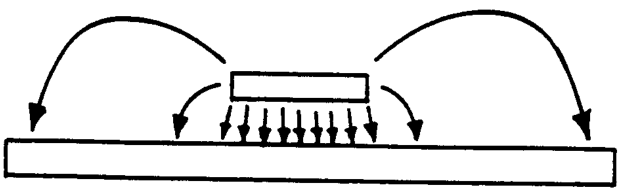


Fig. 2. Section of stripline with sketch of E-field lines

2. Experimental Apparatus

2.1 The **microwave source** for this experiment is based on a product from Analog Devices, Inc., and generates a few mW of continuous wave (CW) microwave power, available from a miniature coaxial SMA panel connector. The generator is entirely microprocessor controlled, and accepts commands to control the frequency, amplitude and other settings by means of a USB interface and a Windows PC.

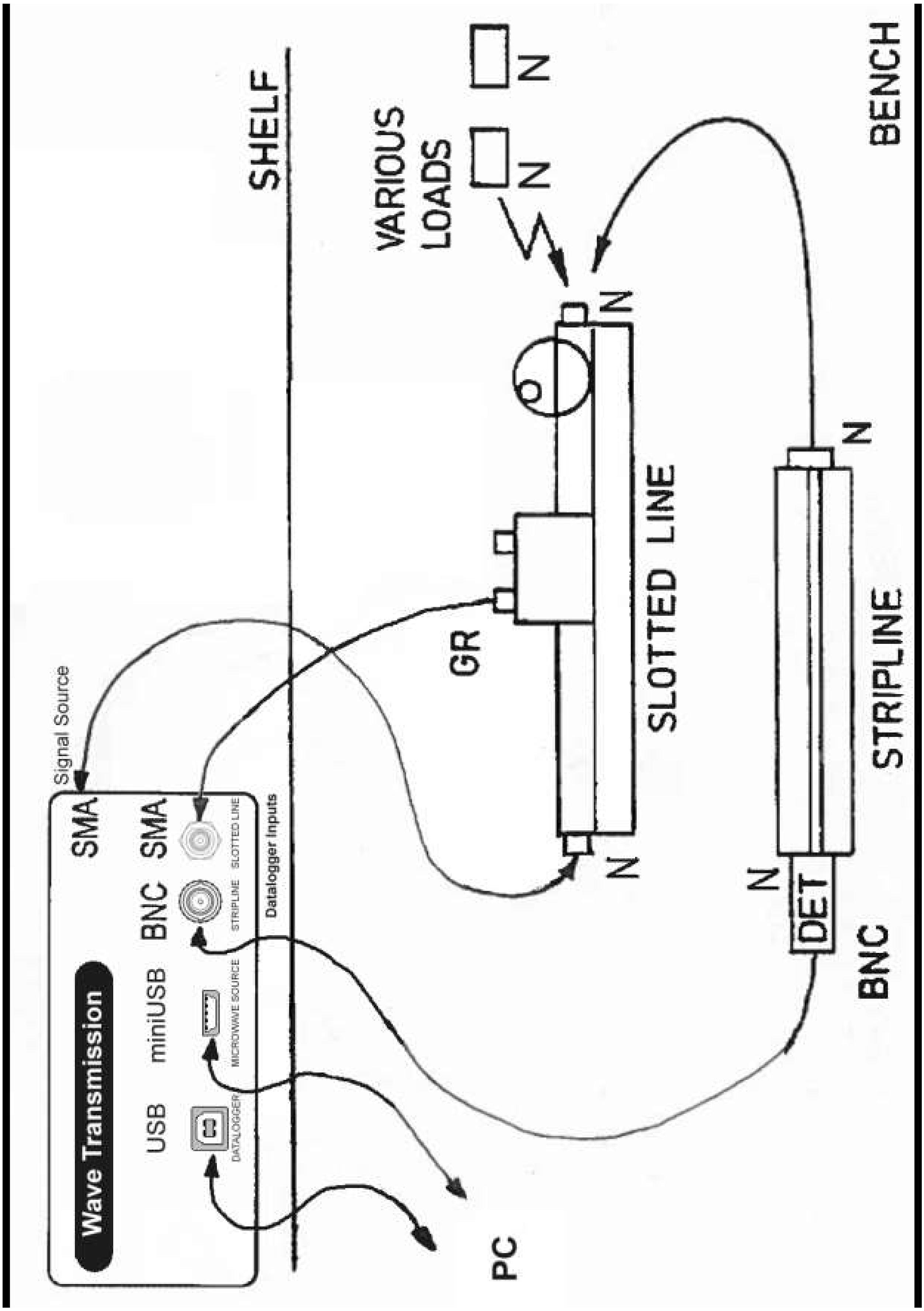
With the Windows application **New Wave** running, the following features are available:

- At top left the Microwave Source panel shows the RF output frequency in MHz, from 2000 to 4000. You may click on the calibrations at 2000, 2400, 3000, 3600 or 4000 to set the frequency to that value. Alternatively, click the arrows at either end of the scale to increase or decrease frequency. The mouse scroll wheel may also be used.
- At the right side of the Microwave Source panel you can set the Output Level (O/P level) to one of four different values, or OFF.
- At the left side of the Microwave Source panel you can click the large rectangular Scan button to command a scan in which the frequency will be swept repeatedly from 2000 to 4000 MHz in 10 MHz increments. This is handy for quick observations.

2.3 **Slotted line**. A fine probe can be moved along the slot using the cranked wheel at the RH end, to allow observation of the relative strength of the E-field between the centre and outer conductor of the rigid coaxial line. The RH connector should have a shorting plug. There is a diode detector in the moveable carriage; it rectifies the alternating E-field and gives a voltage output at the left hand GR connector on the carriage. Please do not attempt to dismantle the carriage assembly. Various loads may be connected to the end of the slotted line: open circuit, short circuit, and 'matched' load, or it may be followed by the next item -

2.4 **Stripline**. This 'open plan' line allows different materials to be introduced into the wave field space between the suspended live conductor and the ground plane, to explore how they affect its properties. At its far end, another diode detector, type CD51, is provided. This will be used to observe the forward wave transmitted through the obstacles to be introduced on the line. With the stripline (unlike the coaxial line), weak fields spread out to some distance either side of the space between the conductors and your observations will be affected by any objects which may be too close: see Fig. 2.

2.5 **A data logger** is also provided and is integrated into the same enclosure as the microwave source. This allows signal levels to be measured numerically and under program control. The **Slotted line** panel is always updated, and records the signal level from the detector attached to the Slotted line detector as a function of time. The Stripline panel is updated when a sweep is in progress, and the colour trace plotted may be selected manually as Blue Trace, Red Trace or Green Trace by clicking the corresponding button at top right. Graphs may also be plotted as a function of frequency. These features will be required in later parts of the experiment. In addition, **coaxial cables** are provided, fitted with the different **connectors** required: type SMA for the microwave source, type BNC for the data logger, and type N for the stripline and for the slotted line and its various loads.


Wave Transmission
USB
miniUSB
BNC
SMA
SMA
Signal Source
DATALOGGER
MICROWAVE SOURCE
STRIPLINE
SLOTTED LINE
Datalogger Inputs
PC
SHELF
GR
SLOTTED LINE
VARIOUS LOADS
N
N
N
N
N
N
DET
BNC
STRIPLINE
BENCH

3. Experiments to be performed

Fig. 3 shows the main components in a convenient arrangement on the bench.

3.1 Measurement of wavelength and phase velocity on the slotted line. Observations of standing waves.

With the equipment powered up and the New Wave application running, set the frequency on the source. You should use three different frequencies, approximately 2.4, 3.0, and 3.6 GHz for this experiment. Make sure the **O/P level** is set to maximum. The output should be connected to the LH end of the slotted line so that the forward wave is in the direction of increasing *z* as measured on the mm scale. Place a short circuit on the right-hand end of the slotted line (it is an N type connector plugged with a brass disc). The detector output on the moveable carriage can now be observed on the PC screen in the **Slotted line** panel. The scale is logarithmic, so variations can most easily be detected at low signal levels of a few mV. The power of the source and the sensitivity of the detector both vary slightly with frequency setting. Vary the frequency slightly, in each of the three cases, to one that gives a strong upward deflection on the PC screen.

Observe positions of minima in the standing wave pattern. Note that the minima are very deep, close to zero, because the wave reflected from the short circuit is equal in amplitude to the forward wave. The logarithmic scale on the PC display will facilitate this, and you may see low-level fluctuations (noise) superimposed. Record the position P_1 of the minimum nearest to the low end of the scale and another Q_1 near the high end of the scale*, counting the number of maxima in between. Hence, from equation (10) and the paragraph which follows it, calculate the wavelength on the slotted line and then the velocity $u = f\lambda$. Obtain values at different frequencies; they should be equal to c (2.998×10^8 ms^{-1}) to very good accuracy.

Headings for your note book. **Table 1**.

Freq.	Position of Minima		Number of max.	λ	u
f/GHz	P_1 / mm	Q_1 / mm	P_1 to Q_1	mm	mm/ns

In all experimental work, calculate results as you go along, and if possible plot them on a graph, in order to verify that they make sense. In this section set out the results in your notebook in a Table with the headings given above. The *accuracy* of the *u*-values should be derived in your Table from a clearly stated estimate of the uncertainty in a single position measurement (what sets the limit to this?), followed by a comment on the consistency of the several *u*-values and their relation to *c*. To estimate the accuracy, refer to the Appendix: *Estimation of Experimental Errors*, attached at the end of this sheet.

* Not *too* near this end. Make sure there is sufficient free travel to reach the next maximum.

3.2 Amplitude reflection coefficients measured with the slotted line

You have already seen that the reflection coefficient of the brass shorting plug is unity: in optical terms it is a perfect reflector. In fact, the reflection coefficient $\rho = -1$ because the reflected wave must be in anti-phase with the incident wave at the brass surface where there can be no tangential electric field.

What would you expect for an open circuit? You might suppose that the wave would carry on into empty space. However, being constrained by the currents in the conductors (which must be zero at an open circuit) it is almost totally reflected – just as the sound wave in an organ pipe is mainly reflected at the open end of the pipe.

One frequency will do for this experiment. Choose one from your Table 1 near 3 GHz and set the same frequency. Remove the brass shorting plug, and connect instead the special 'open' plug – which is of the same length as the shorting plug. Measure the new positions of previous minima which will have now shifted to P_2 and Q_2, with $P_2 > P_1$ and $Q_2 > Q_1$ (this is the reason that Q_1 was specified as not too near the high end of the scale). Comment on the depth of the minima observed. Use the following table headings in your notebook and the same method as previously for assessment of accuracy.

Table 2

f	P_2	$(P_2 - P_1)$	Q_2	$(Q_2 - Q_1)$	Average Shift	Δ/λ
GHz	mm	mm	mm	mm	Δ mm	

What shift Δ/λ do you expect? If your result is different, what does this imply? Give a quantitative answer.

You might expect there to be some characteristic impedance, in between a short circuit and an open circuit, with *zero reflection coefficient* i.e. total absorption of the incident wave. Indeed there is, but since it is difficult to assess that case with this apparatus you will now measure an intermediate reflection coefficient.

Connect a flexible cable from the slotted line to the stripline and place one brass screw as a short circuit between the suspended conductor and the ground plane near the LH end. Screw down lightly; if you screw down hard you will distort the line. Keep other objects well away from the stripline fields. The reflection coefficient ρ is defined in terms of the relative magnitudes of the forward and reflected waves A and B – see Equation 7; however, these are difficult to measure directly. Instead we define a quantity *S*, called the *Standing Wave Ratio*, which is related to ρ, but can be determined from direct measurement of the fields on the slotted line. *S* is defined as the ratio of the field magnitudes measured at the maximum and minimum positions on the slotted line. These magnitudes depend on A and B – a moment's consideration of section 1.6 shows that $(A + B)$ corresponds to the maximum observed amplitude, and $(A - B)$ to the minimum observed amplitude. Hence:

$$S = \frac{A+B}{A-B}, \qquad \text{and by re-arranging,} \qquad \rho = \frac{B}{A} = \frac{S-1}{S+1}, \qquad (11)$$

using the notation presented on page 4.

Unfortunately the detector output is offset from zero, and it is non-linear (it is in fact proportional to the *square* of the signal value). To work around the first problem, record the lowest value observed, which represents the baseline of the plot. Subtract this value from all subsequent measurements to get a proper zero-reference value before substituting in the above equations. To take into account the second problem, you should assume that the ratio between maximum and minimum that you observe represents S^2.

Measure S^2 with the brass screw in position and hence deduce the reflection coefficient, ρ. It clearly does not behave as a perfect short circuit.

4. Velocity of microwaves in different materials

In this section we shall investigate the effect of placing different dielectric materials in the space between the stripline and the ground plane. This affects the velocity of propagation of the microwaves and hence the wavelength.

Retaining one screw in the stripline, start Table 3 with the headings given below and record the position M of a minimum on the slotted line, somewhere near the LH end.

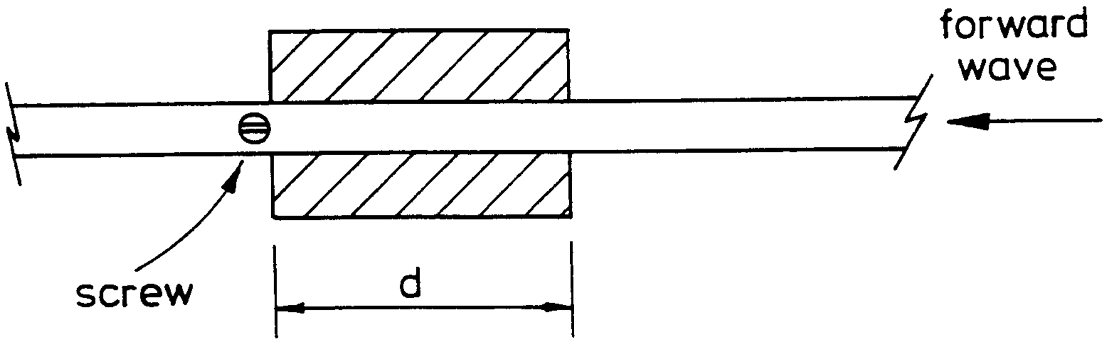


Fig. 4

Now introduce a sheet of material 3 mm thick under the suspended line, up against the reflecting screw as shown in Fig. 4. Materials are provided with values of *d* from 30 mm to 120 mm: begin with the shortest samples.

The reduced velocity in the material increases the effective total length of the line to the reflector – the quantity *l* in equation (10). Hence the position of each minimum on the slotted line moves nearer to the reflector by an amount (n-1) d. Record the new position *R*, and interpret the shift in terms of refractive index, *n*.

Remember the earlier advice to do calculations as you go along. If *d* is large you may be at risk of making a mistake about which minimum is which. To avoid this risk, consider gradually introducing the sample corner-first, tracking the chosen minimum with the detector. Ask a demonstrator if you are in doubt.

Table 3

Material	*M*/mm	*d*/mm	R/mm	*R*-*M*/mm	*n*

Use four different *d*-values and three different materials: teflon, perspex and glass. Assess the accuracy of the separate results as before and comment on the internal consistency. Refractive index values at microwave frequencies may be greater than you expect from their value at optical frequencies.

5. Transmission through a Resonator

Any resonant system, such as a violin string, can have one particular mode frequency (and harmonics) defined by the distance between "stops" or amplitude minima. The stripline has provision for 2 brass screws to be inserted with marked spacings. These are highly reflecting (but not totally reflecting, as you should have noticed in section 3.2). Consequently wave energy oscillates to and fro between these two reflectors. Resonance is reached when the spacing between them is an integer number of half wavelengths (not *quite* exactly, as it turns out). There is transmission through the system at resonance when successive reflections add in phase.

A spacing of 120 mm between the screws will produce a single resonance between 2 and 3 GHz, and this is explored first.

5.1 Setting up the apparatus

(a) Remove from the stripline the screw and materials used in the previous part of the experiment. Check that the lead from the detector on the end of the stripline is connected to the Datalogger interface. (See Fig 3, p 6).

(b) Select the required colour (blue, red or green) for the datalogger trace by clicking the appropriate button in the **Datalogger panel**. Enter any comment or note you wish to accompany the printed plot in the **Add a note ..** text box below

5.2 Measurement of a single resonance

(a) Watch the stripline Click the Scan button in the **Microwave Source** panel to get a fast scan, and observe the effects. It will take about 6-8 seconds for the whole 2 to 4 GHz sweep, which in Scan mode is sampled at intervals of 10MHz . It should be noted that the screen plot baseline may move up and down to some extent as the generator output power varies with frequency. By adjusting the power level on the Microwave Source panel, the display can be set to a suitable position. Watch one or two sweeps, until you are happy with the settings. This is a Reference plot, with which you should compare later results.

(b) Record the Reference plot, but clicking the **Log .. trace** button in the **Datalogger** panel. A slower sweep will take place with a frequency increment of 2MHz per point, and the results will be plotted in the chosen colour. Click the **Log .. trace** button a second time to stop the sweep. You may observe a superimposed ripple effect on certain equipment where minor impedance mismatches remain (e.g. in the connectors). If so, you may find it worth trying the **FILTER** button at top left of the plot area. Click the button to apply the inbuilt filter to all plots on the screen. Click it a second time to disable the filter.

(c) Now insert two screws at a spacing of 120 mm through the stripline so that they lightly but firmly touch the metal base – do not over tighten them or the stripline will be distorted and bowed away from the base. Then keep your hands away from the stripline.

(d) Run a frequency sweep, as described in step (a). Check you have chosen a suitable trace colour for the next plot. Watch the display in the **Stripline** window and when you are ready, click the **Log .. trace** button in the **Datalogger** panel. You can display and print up to 3 traces at one time on the screen.

(e) Adjust the frequency of the Microwave Source using the mouse so that the frequency coincides with the peak resonant output. Read off the frequency from the corresponding box in the Microwave Source panel, and record this carefully.

(f) **Print out the graphics window** by clicking on the **PRINT** button on the screen (top right of plot window). Note that every printed plot is also saved locally. You can use the **OPEN** button to return to earlier datasets accumulated during the session

(g) **Mark the peak frequency** onto the plot and determine what wavelength corresponds to this frequency. Is it 120 mm? Can you suggest why might it be consistently more or less?

5.3 Multiple resonances

If you have time, this is a more interesting result for a resonator formed with two screws spaced at 320 mm on the stripline.

Remove the screws from the stripline to obtain a reference plot again, as per paragraphs 5.2(a) and (b). Insert the screws 320 mm apart – only lightly screw them down, checking each one, and repeat the rest of section 5.2(d) onward.

Using the mouse to adjust the **Microwave Source** frequency, measure the frequencies of the *maxima* seen in the **Stripline** window, making an estimate of what errors each reading might have. Use the filter if necessary to achieve a clean plot. Label your plot and fix it in your lab notebook. Make calculations in tabular form with a line for each resonance (four in all) and with column headings, frequency, wavelength λ, $N = 320\ \text{mm}/\lambda$ and, by rounding N to the nearest half wavelength, deduce the "effective spacing" (in wavelengths) between the screws for each resonance. Comment on these results.

6. Report

As this is a Long Experiment, you will need to write a report. General guidelines about report-writing are to be found in the information you received at the beginning of the year.

An important objective of this practical is to instil good habits in the recording and treatment of results. The emphasis on accuracy is not necessarily to achieve a high accuracy, but to make a realistic, quantitative assessment of the accuracy that can be achieved. Refer to the Appendix on *Estimation of Experimental Errors* (attached) for guidance on how to assess the accuracy of the results you obtain, and how to present them in a consistent way.

For this experiment, you should take care to include:

- Summary: a brief summary of the aims of the experiment
- Readings and results: include all the measurements asked for during the experiment.
- Discussion: discuss the results and the accuracy you have achieved, and answer the questions posed in the text.
- Conclusions: summarise your findings and achievements.

Original Script – Dr P J Spreadbury
Revisions 2006-2013
Dr D M Holburn, December 2015

Experiment E4 – Appendix: Estimation of Experimental Errors

It is good practice in experimental work to make an estimate of the likely error in the result before we actually do the experiment. Although this may sound back-to-front, it is actually a very important step, as it can give us a useful insight into how much confidence we can have in the result, and which parts of the experiment most deserve our attention to improve its accuracy. With this approach we can readily take into account:

1. Instrumental inaccuracies. These may be systematic (due to a faulty zero or scale constant) or random (due to friction or 'noise').
2. Estimates made when fractional parts of scale divisions are read.
3. Numerical rounding-off during calculations.

The estimate is obtained by estimating the magnitudes of the individual errors associated with the items in the list above, and carrying these errors through the numerical processing of the experimental data to give the corresponding error in the result. Below are some simple rules for carrying out these calculations.

An error in some quantity x can be expressed as an absolute error ε (for example, a wire of length 122 ± 0.5 mm), or as a relative error r, where $r = \varepsilon/x$. Often it is convenient to use *percentages* to represent relative errors (for example, a current of 3.25 A ± 1%).

Numerical processing may involve:

(a) **Addition and subtraction**: if two quantities x_1 and x_2 may be in error by as much as $\pm\varepsilon_1$ and $\pm\varepsilon_2$ then the absolute error in the sum or difference of x_1 and x_2 may be as much as $\pm(\varepsilon_1+\varepsilon_2)$ in the result. Note that subtracting two nearly equal qualities can produce a large value for the relative error, $\pm(\varepsilon_1+\varepsilon_2)/(x_1-x_2)$, in the result. You need to look out for such cases in your experimental work.

(b) **Multiplication and division**: if two quantities x_1 and x_2 may be in error by as much as $\pm r_1x_1$ and $\pm r_2x_2$ then the relative error in the product or quotient of x_1 and x_2 may be as much as $\pm(r_1+r_2)$, provided r_1 and r_2 are small.

Note that absolute errors *must* be used when dealing with addition and subtraction: relative errors *must* be used when dealing with products and quotients.

It also follows that where we have to deal with expressions containing quantities raised to a power (e.g. x^2), the relative error in the result, x^2, is twice the relative error in x. More generally, where x must be raised to the power P, the relative error in the result x^P is P times the relative error in x. This also holds for fractional powers, i.e. the relative error in the square root of x is one half the relative error in x.

The above ideas are usually quite sufficient for laboratory experiments. However, it may be argued that if x_1 and x_2 are independent variables, it is unlikely that the largest errors in both quantities will occur simultaneously.

It can be shown that if the probable absolute errors in x_1 and x_2 are $\pm\varepsilon_1$ and $\pm\varepsilon_2$, then the probable absolute error in the sum or the difference of x_1 and x_2 is $\pm\sqrt{\left(\varepsilon_1^2+\varepsilon_2^2\right)}$.

Similarly, if the probable relative errors in x_1 and x_2 are $\pm r_1$ and $\pm r_2$, then the probable relative error in the product or quotient of x_1 and x_2 is $\pm\sqrt{\left(r_1^2+r_2^2\right)}$.

In this experiment you are advised to use the simple approaches outlined above, and not to worry about these refinements.

Simple examples

The following are a couple of simple examples that illustrate how to apply the ideas of the first section.

(a) The constant acceleration a of a slowly moving object is to be found by determining the time t taken to traverse a measured distance s . The equation of motion that applies is:

$$s = \frac{1}{2}at^2. \text{ Rearranging, } a = \frac{2s}{t^2}.$$

The time is measured with a stopwatch, the distance with a metre ruler. The measured values and their errors are:-

$s = 2 \pm 0.005$ m. This is 0.25 %.
$t = 4.2 \pm 0.2$ s. This is 4.8 %.

What is the acceleration a and its estimated error?

The relative errors in a and t^2 may be added to give the relative error in a. The relative error in t^2 is twice the relative error in t.

Hence relative error in a is: $0.25\% + 2 \times 4.8\%$ = **9.8 %**.

The factor of 2 in the time term causes that term to dominate. The ¼ per cent error due to the distance measurement is clearly negligible compared to the 9.6% error due to the time measurement, so the result (the acceleration) would most sensibly be written:

$$a = 0.23 \pm 0.02 \text{ m s}^{-2}.$$

(b) An inductor is formed by winding N_1 turns of wire in the form of a cylindrical coil of length l and diameter d_1. A second circular coil with N_2 turns and of smaller diameter d_2 is placed coaxially within the first, at its centre. The mutual inductance between the two windings is to be determined from measurements on the two coils, using the formula:-

$$M = \frac{\pi\mu_0}{4}\frac{N_1 N_2 d_2^2}{\sqrt{d_1^2 + l^2}} \text{ H}$$

The numbers of turns were counted and are exact. Lengths were measured with a metre rule; the measured values and their errors (percentages quoted in parentheses) are:-

N_1 = 200 turns (exact)	$d_1 = 20 \pm 0.5$ mm (2.5 %)
N_2 = 20 turns (exact)	$d_2 = 10 \pm 0.5$ mm (5 %)
$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}	$l = 50 \pm 0.5$ mm (1 %)

What is the mutual inductance M and its expected error?

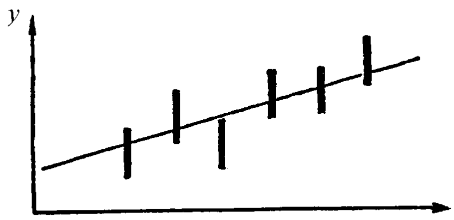
The relative error in the numerator due to d_2^2 is: 2 × 5 = 10%. Relative errors in d_1^2 and l^2 are 5% and 2% respectively, so the absolute values are: $d_1^2 = 400 \pm 20$ mm^2 and: $l^2 = 2500 \pm 50$ mm^2, and the value of the term inside the square root is: 2900 ± 70 mm^2.

The relative error in the denominator is: ½ × 70/2900, or 1.2%, and the overall relative error is **10% + 1.2%** = **11.2** %. It can be seen that the most serious error here arises from poor accuracy in measurement of d_2. The result might therefore be written:

$$M = 7.3 \pm 0.8 \times 10^{-6} \text{ H}$$

Stating results and plotting graphs

In a number of places in this or other experiments, you may be asked to find the value of some key experimental parameter from measurements made. When quoting the result, it is important to bear in mind the uncertainty or likely error in the quoted value. For example, it is meaningless and misleading to quote a result to 10 significant figures (although your calculator may work out the figures to this precision) if the error is, say, a few per cent, which may be typical of experiments done in the Part IB laboratory. In this case a result quoted to two or perhaps three significant figures might be appropriate; and the magnitude of the likely error should always be presented. Study the examples (a) and (b) on the previous page for illustrations of accepted practice when presenting results.



In some cases you may be asked to find the relationship of some quantity y to another quantity x.

Plotting your results as precise points in the x, y plane leads to the problem of finding the 'best' straight line (or higher-order polynomial) fit to the points. The method of least squares, described in the mathematics course, provides a solution to this problem.

However, it may be more practical to include estimates of the experimental errors at the plotting stage by plotting your results as rectangles of height equivalent to the calculated error, as shown opposite. If you can draw a straight line through at least 2/3 of the rectangles, then your experiment is consistent with the hypothesis that y is a linear function of x. Wherever possible you should try and plot data in this way *as you make the measurements* – this approach will often warn you instantly if you make an inadvertent error of measurement. It may be difficult to track down such errors once you have left the laboratory.

Some programmable calculators have a built-in routine for finding the best straight-line fit to a set of points. However, the hypothesis that one variable in your experiment is linearly dependent on another is something that should come from engineering science, not from the experiment itself.

Plot the points first, to see if the relationship is a linear one. Then use your calculator, if you want to, to find the numerical values of the parameters of the line.